SALVE WINTER SCHOOL PARIS, DECEMBER 6–7–8 2023

The conference will take place at the École normale supérieure (ENS), Département de Mathématiques et Applications (DMA).

The lectures will all be given in **Salle W**. The welcome coffee, coffee breaks and lunch buffets will be in **Espace Cartan**. (See the map on last page of this document.)

There will be a reception at the 24th floor of the Zamansky Tower on Thursday evening. All participants are invited.

SCHEDULE

Wednesday (Dec 6)

8:30–9:00 Welcome coffee

9:00-10:30 Chiara Saffirio

Weakly interacting fermions: mean-field and semiclassical regimes (1/3)

10:30-11:00 Coffee break

11:00–12:30 Léo Bigorgne Asymptotic behavior of the the solutions to the Vlasov-Maxwell system with a small distribution function (1/3)

12:30-14:00 Lunch buffet

14:00-14:30 Coffee

14:30–16:00 Diogo Arsénio Hydrodynamic regimes near Fermionic condensates $\left(1/3\right)$

16:00-16:30 Coffee break

16:30–17:30 Mahir Hadzic Stability, oscillations, and damping for the gravitational Vlasov-Poisson system (1/3)

Thursday (Dec 7)

9:00–10:30 Mahir Hadzic Stability, oscillations, and damping for the gravitational Vlasov-Poisson system (2/3)

10:30–11:00 Coffee break

11:00–12:30 Léo Bigorgne Asymptotic behavior of the the solutions to the Vlasov-Maxwell system with a small distribution function (2/3)

12:30-14:00 Lunch buffet

14:00–14:30 Coffee

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14:30–16:00 *Diogo Arsénio* Hydrodynamic regimes near Fermionic condensates (2/3)

16:00–16:30 Coffee break

16:30–17:30 Chiara Saffirio Weakly interacting fermions: mean-field and semiclassical regimes (2/3)

19:00–21:30 Reception at Zamansky Tower, Jussieu campus (Sorbonne Université)

Friday (Dec 8)

9:00–10:30 Chiara Saffirio Weakly interacting fermions: mean-field and semiclassical regimes (3/3)

10:30–11:00 Coffee break

11:00–12:30 Mahir Hadzic Stability, oscillations, and damping for the gravitational Vlasov-Poisson system (3/3)

12:30-14:00 Lunch buffet

 $14{:}00{-}14{:}30\ Coffee$

14:30–15:30 Léo Bigorgne Asymptotic behavior of the the solutions to the Vlasov-Maxwell system with a small distribution function (3/3)

15:30–16:30 *Diogo Arsénio* Hydrodynamic regimes near Fermionic condensates (3/3)

ABSTRACTS

Diogo Arsénio

Hydrodynamic regimes near Fermionic condensates

A Fermionic condensate is a superfluid state of matter formed by fermions at very low temperatures. In the Boltzmann–Fermi–Dirac equation, which governs the evolution of a Fermi gas, this superfluid phase is identified by an equilibrium state in which all particles occupy their lowest possible energy state, while obeying the Pauli exclusion principle. Our goal is to explore hydrodynamic regimes of the Boltzmann–Fermi–Dirac equation near a Fermionic condensate. In particular, we will discuss how the control of the relative entropy near absolute zero leads to singular velocity distributions which are concentrated on a sphere. The geometry of collisions in this quantized state will require special care and some new tools. We will also show how the analysis of the relative entropy near Fermionic condensates allows us to establish some hydrodynamic regimes of the Boltzmann–Fermi–Dirac equation, while opening doors to fresh new research perspectives.

Léo Bigorgne

Asymptotic behavior of the the solutions to the Vlasov-Maxwell system with a small distribution function

We will be interested in the solutions to the Vlasov-Maxwell system arising from sufficiently small and regular data. In particular, we will compare their asymptotic behavior with the ones of the solutions to the corresponding linear equations. Even if the electromagnetic has a nontrivial memory effect, it enjoys linear scattering since it approaches, for large time, a solution to the vacuum Maxwell equations. In contrast, the distribution function merely satisfies a modified scattering statement. Due to the long-range effects of the Lorentz force, it converges along logarithmic corrections of the linear characteristics. In order to define these modified characteristics, a key step consists in identifying an effective Lorentz force governing the asymptotic behavior of the force field.

Mahir Hadzic

Stability, oscillations, and damping for the gravitational Vlasov-Poisson system

The gravitational Vlasov-Poisson system is a classical astrophysics model describing the dynamics of galaxies. It allows for a vast amount of steady states, whose stability is a question of central importance in the field.

In this mini course we shall start by first reviewing the orbital stability theory for compactly supported self-gravitating steady galaxies. These ideas were initiated in the seminal works of Antonov in 1960s, which led to a conjecture that those steady states with a decreasing number of ever more energetic stars in their support are stable. We shall discuss the methodology and a rigorous resolution of this conjecture from 2000s.

The orbital stability results do not tell us much about the asymptotic stability of the solutions. The second half of the course is devoted to a

very recent progress on this question, which is also known as the problem of gravitational relaxation a.k.a. the gravitational Landau damping. This topic was initiated in the physics literature in a series of seminal papers by Lynden-Bell from 1960s, however the first rigorous results are very recent. We shall highlight the use of spectral methods, including the Birman-Schwinger principle, gravitational phase-mixing, and ideas from scattering theory.

Chiara Saffirio

Weakly interacting fermions: mean-field and semiclassical regimes The derivation of effective macroscopic theories approximating microscopic systems of interacting particles is a major question in non-equilibrium statistical mechanics. In this course we will be concerned with the dynamics of systems made of many interacting fermions. We will focus on the meanfield regime, i.e. weakly interacting particles whose collective effect can be approximated by an averaged potential in convolution form, and review recent mean-field techniques based on second quantization approaches. As a first step we will obtain a reduced description given by the time-dependent Hartree-Fock equation. As a second step we will look at longer time scales where a semiclassical description starts to be relevant and approximate the many-body dynamics with the Vlasov equation, which describes the evolution of the effective probability density of particles on the one particle phase space. The structure of the initial data will play an important role at each step of the approximation.

